Speed, optical power, and off-axis imaging improvement of refractive liquid crystal lenses

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Two design approaches (multicell and addition of phase resets in single cell) are introduced to optimize the performances of tunable refractive liquid crystal lenses, including improvements on the switching speed, optical power, and the off-axis, wide-angle imaging performance. Key parameters and advantages for each method are discussed, and their effects on the performance are demonstrated in detail with numerical calculations. © 2014 Optical Society of America

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1. Introduction

Liquid crystal (LC) lenses have long been studied with their advantages over conventional lenses such as small size, tunable focus with no moving parts, high-speed switching, light weight, etc. A great number of designs and approaches have been reported, such as patterned electrode-based phase control by electric field [1–12], polymer-based lenses [13–15], application of blue-phase LCs [16,17], patterned-relief-structure-based lens [18], hybrid lens [19], etc. Most of them could be classified into refractive or diffractive lenses, each of which has its own advantages and disadvantages. Obviously, the optical tunable range, switching speed, and chromatic performance are very important concerns in broad applications. Usually, the diffractive lenses are able to achieve high power and fast speed by having a very thin LC layer. However, they have very significant chromatic aberrations, and the performance for non-design wavelengths of the light are dramatically degraded. On the other hand, the refractive lenses have much better chromatic performance, but usually they tend to have a trade-off between the large optical tunable range and the fast switching speed, i.e., large optical power can be provided by thicker cells but the response time would be dramatically increased.

LC lenses based on a ring electrode structure with very high quality performance and in-depth analysis have been shown [20–22]. Therefore, in this work, the motivation is to consider further improving the tunable power and switching speed of the lens design, as well as large-angle imaging performance.

The first method is the multicell approach, here using a pair of cells to double the optical power of the lenses. The switching speed of this dual-cell device with higher power is the same as its single lens component, while the switching speed is four times that of a single lens of the same power as the dual-cell device (response time is proportional to the square of the LC layer thickness). While this design feature is obvious, a significant point of this work is the implementation of this idea when using a lens design with ring electrodes. As directly stacking
two identical lenses causes a lower efficiency because the actual phase sampling rate (phase steps per wave) is cut to half of that for the single lens, we demonstrate that this degradation can be eliminated by the proper choice of electrode geometry for each component lens of the stack. In addition, we discuss the problems of large-angle imaging of LC lenses in detail, which have barely been covered previously; and a special arrangement of dual-lens stacking, as an advantageous effect of the multicell design, is demonstrated to improve the off-axis imaging performance of the LC lens.

The second approach is to adopt the advantages of both refractive and diffractive lenses. A much more balanced performance can be obtained by adding a smaller number of phase resets to a typical refractive lens, which works like a Fresnel lens with few subzones. As a result, the optical power is increased while the switching speed remains by keeping the same cell thickness, and the performance for nondesign wavelengths is not significantly degraded. We will discuss the effects of several parameters, such as number of resets and magnitude of resets, both analytically and numerically.

2. Numerical Approach

The calculation used in this work includes a 2D LC director field and optical lens modeling. The LC director field calculation takes the electrode pattern, applied voltages, cell thickness, and the LC properties as input, and numerically calculates the 2D director profile throughout the cell. Particularly, the property parameters of the LC material (liquid crystal mixture 18,349) used in the modeling are $K_{11} = 11.0 \times 10^{-12}$, $K_{22} = 7.1 \times 10^{-12}$, $K_{33} = 30.5 \times 10^{-12}$, $n_e = 1.8$, $n_o = 1.53$, $\Delta n = 0.27$ ($\lambda = 589$ nm, 20°C), and $\epsilon_\parallel = 21.9$, $\epsilon_\perp = 6.1$ (1 kHz, 20°C). Then the optical path difference (OPD) or phase profile of the lens can be calculated by integrating the effective refractive index across the cell thickness for each point on the lens surface. With a given lens OPD, lens modeling is used to calculate the light distribution in the focal plane based on the wave nature of the light. For the purpose of obtaining the most accurate results, a Rayleigh–Sommerfeld propagator will be used [23,24]. The optical efficiency of the lens is considered as a Strehl ratio, which is the ratio of the peak intensity in the focal plane for LC lens over the ideal thin lens with parabolic phase profile. Also, the intensity distribution in the focal plane can be considered as a point spread function (PSF), from which modulation transfer function (MTF) can be obtained by taking the Fourier transform [23,24]. The detailed numerical considerations and procedures can be found in our previous publication [25].

3. Multicell Approach

A. Stacking of Lenses with Different Electrode Geometry

Generally speaking, stacking two identical ideal thin lenses exactly on top of each other would double the optical power, while the phase profile would stay as an ideal parabola. As a result, the optical performance would have no difference from a single ideal lens with the half focal length. However, the case is different for actual LC lenses with discrete ring electrodes because the phase profile is sampled to steps, and the optical efficiency is a function of the discretization of the continuous profile [26]. If two identical LC lenses are attached with their electrode structure exactly on top of each other, and no separation between them is considered, the optical power can be doubled with the basic shape of the total phase profile being parabolic. However, the effective number of ring electrodes across the aperture stays the same, meaning the resultant sampling rate (phase steps per wave) becomes only half that of the single lens, and the efficiency would be greatly reduced.

Therefore, a dominating factor for the efficiency is the effective sampling rate across the lens aperture. To solve this problem, a slightly new electrode ring mask design is needed to make a lens stack with the current design, to provide a higher sampling rate across the aperture. To obtain the exact dimensions for electrodes on the new lens, we can start from the objective phase profile and the expected phase sampling of the stack; phase stages are used for demonstrating the discrete phase steps, and no smoothness between steps is considered.

We have shown that when the objective focal length, lens diameter, and the sampling rate of the phase profile (number of steps per wave) are known, the number and the dimension of ring electrodes in need are obtained. The phase profile of an ideal thin positive lens is approximately a parabola, expressed below [27]:

$$\text{OPD}(r) \approx -\frac{r^2}{2f}.$$

Here, $r$ is the lens radius and $f$ is the focal length. If the sampling rate is $f_s$ (number of phase steps per wave) and the area of each ring electrode is equal, the total number of rings is obtained as $N = \text{OPD} \cdot f_s / \lambda$. For example, if the focal length is $f = 400$ mm, design wavelength $\lambda = 543.5$ nm, and a sampling rate of 10 phase steps per wave is considered, it would require 33 electrodes in total, where the outer radius of each ring can be expressed as

$$r_n = \sqrt{\frac{2nf}{f_s}}, \quad n = 1, 2, \ldots, N.$$

Here, $n$ is the index number of each ring electrode. Now consider that we would like to have a composite lens that is fabricated as a stack of $M$ component lenses. The number of phase steps of the composite lens should have a value $N$ determined by the target for its efficiency. It might be considered that each component lens needs to have $N$ ring electrodes, but we will show in this work that this is not
required, and each component lens is only required to have \(N/M\) ring electrodes if their geometry is done correctly.

The proposed correct geometry is to have the radii of the component lenses to be staggered so that the composite lens has a resulting phase profile that has \(N\) equal-amplitude phase steps. As an example, consider the case where the composite lens is fabricated from two component lenses \((M = 2)\), and where the effective composite lens has \(N\) phase steps located at radii determined by Eq. (2). The correct geometry for the rings of the two component lenses should be such that the radii of the electrodes of the first component lens will be specified by the even values of \(n\) in Eq. (2), and the radii of the electrodes in the second component lens will be specified by the odd values of \(n\).

Figure 1 shows the phase profile of two component lenses and the resulting phase profile of the composite lens for the case of a target composite lens having values of \(f = 200\) mm and \(f_s = 10\).

Based on this design, we calculate the PSF of the lens using the previously introduced method [25]. The Strehl ratio is calculated as 96.6\% (normalized to the ideal lens with \(f = 200\) mm) for the composite lens, assuming stairs with no smoothness between steps in the phase profile. However, if the second component lens is identical to the first, the stair profile has only five phase steps per wave, and the Strehl ratio calculated in focal plane \(z = 200\) mm is only 87.36\% of the ideal lens. When we use a full numerical model of this two-lens system, the efficiency is calculated as 98.5\% because of the smoothness of the edges in the profile caused by the fringing field.

More generally, if the total number of phase steps needed for the composite lens is \(N\), and the stack consists of a certain number of component lenses given as \(M\), the geometry of the electrode pattern on each component lens can be obtained. Assuming that \(N/M\) is an integer, a simple method can be used. The radii of the phase steps of the composite lens can be determined using Eq. (2), and then the first component lens would have electrodes with outer radii the same as every \(M\)th phase step of the composite lens starting from the first phase step of the composite lens; the electrodes of the second component lens are defined by every \(M\)th phase step of the composite lens starting from the second phase step of the composite lens, etc. Therefore, based on Eq. (2), the expression for the outer radius of each electrode on each lens can be derived as

\[
r_{m,n} = \sqrt{\frac{2sf(m + M(n - 1))}{f_s}},
\]

\(m = 1, 2, ..., M, n = 1, 2, ..., N/M.\) (3)

Here, \(f\) is the objective focal length of the composite lens, \(f_s\) is its sampling rate, \(n\) is the index number of the electrode from the lens center of each component lens, and \(m\) is the index number of each component lens. In addition, since the last electrode of the \(M\)th lens is defined exactly by the outermost radius of the composite lens \((R)\), its total number of electrodes is \(N/M\). However, for the first to \((M - 1)\)th lenses, the last ring defined from the stack is the second to last of its electrode pattern. Therefore, the outermost electrode should be added with its outer radius equal to the composite lens radius \(R\), and there are \((N/M + 1)\) electrodes, in total, in the pattern of these lenses:

\[
r_{m,N/M}\!+\!1 = R, m = 1, 2, ..., M - 1.\) (4)

With this method, the composite lens will have the same sampling rate as its component lenses when they are used individually, and this has the advantage that the switching speed remains fast and the optical efficiency remains high, with minimal chromatic aberrations.

B. Off-axis Large Angle Imaging Performance and the Improvement

Related to the use of multiple-component lenses, to fabricate a composite lens, an improvement in off-axis light performance can be obtained by following certain guidelines. It is well known that the off-axis performance for LC lenses is greatly dependent on the oblique angle of the incident light, as the birefringence of LC material is inherently a function of the angle of light polarization with respect to its director. Even though the phase profile for on-axis light is optimized as a parabola, the off-axis light sees a deviated OPD profile when it passes through LC lenses.

Assuming an azimuthal orientation direction in the cell is defined by the rubbing axis \([x\) axis in Fig. 2(a)], and the absorptive axis of the linear polarizer is along the \(y\) axis [Fig. 2(a)], the on-axis light should propagate in the \(z\) axis with its polarization direction in the \(x\) axis. In the case of light with an oblique angle with respect to the lens plane, there are generally two fundamental cases with different light propagation and polarization directions with respect to the rubbing: (1) the propagation direction of

Fig. 1. Phase profiles of two component LC lenses and the resulting composite lens.
the off-axis light and the polarization are in the \(x-z\) plane; (2) the propagation direction of the off-axis light is in the \(y-z\) plane while the polarization of the light is in the \(x\) axis.

Concerning the first case, there are two directions that the oblique light can be incident on the lens surface with positive and negative angles, having different angles with respect to the tilted LC director axis [Fig 2(b)]. As an example, a LC lens can be modeled with a focal length \(f = 400\) mm, a diameter of 2.4 mm, a thickness of 10 \(\mu\)m, a sampling rate of 10 phase steps per wave, and 1 \(\mu\)m electrode gaps; the voltage profile is optimized for a perfectly parabolic phase profile for on-axis incident light. Once the director orientation of the LC lens is obtained, the effective OPD of the LC lens as a function of the angle of off-axis light can be calculated based on the Extended Jones method [28], from which the light distribution in the focal plane is calculated.

It’s shown that the OPD across the lens aperture starts to deviate from ideal parabolic profile as the angle of the off-axis light increases, and the basic shape of the profile is not parabolic, which induces aberrations and affects LC lens wide-angle performance (Fig. 3). The Strehl ratio and MTF are both calculated for the case of 20° off-axis light, indicating significant image degradation with large-angle light (Table 1). The results show that the Strehl ratio drops to 71.32% at a positive 20° off-axis light, normalized to the peak intensity for an ideal lens (\(f = 400\) mm) in its focal plane with on-axis light, and the MTF significantly deviates from the ideal case. Moreover, with a negative 20° off-axis light, the efficiency becomes even worse as the Strehl ratio drops further to 46.17%, and the MTF shows a much greater drop (Fig. 4). Therefore, this nonsymmetry of the angle dependence of the performance makes the problem more complicated.

For the second case, when the off-axis light is in the \(y-z\) plane and the polarization is in the \(x\) axis, the polarization of the light is always the same as the rubbing direction, which is independent of the oblique angle of the light [Fig. 2(c)]. As the light goes off-axis, the polarization of the light and its angle with respect to the directors remain the same. As a result, in this off-axis light condition, the effective refractive index of the LC is same as that for on-axis light, and the optical performance is independent of the off-axis angle. The Strehl ratio is calculated as 95.4%, normalized to the peak intensity for the ideal lens. The MTF is calculated slightly lower than ideal lens case (Fig. 4).

In addition, the light distribution is calculated for an ideal lens with a continuous parabolic phase profile (\(f = 400\) mm, diameter \(d = 2.4\) mm) in its focal plane with on-axis and 20° off-axis light, and it shows that for the off-axis light the Strehl ratio drops to 97%, normalized to that for on-axis light in the same focal plane, and the MTF shows almost no deviation from the ideal diffraction-limited case. Therefore, for such a lens with a small aperture, the aberration due to off-axis light itself is ignorable; the efficiency drop of LC lenses for off-axis light is mainly caused by the LC material's angular dependence.

Therefore, we propose a simple approach to minimize the phase aberrations for the off-axis light condition: dual lenses with antiparallel alignment. When the off-axis light passes through dual lenses, the phase deviation caused by one lens is greatly compensated for by the other, and the phase profile aberrations are greatly minimized. This arrangement not only improves the off-axis performance, but also gives double optical power and four times faster switching time as the advantages of stacking.

![Fig. 2. (a) LC lens diagram in 3D coordinate, (b) cross section of lens thickness in the \(x-z\) plane and along the radial axis in the \(x\) direction, and (c) cross section of lens thickness in the \(y-z\) plane and along the radial axis in the \(y\) direction.](image)

![Fig. 3. Calculated OPD versus off-axis incident light.](image)

**Table 1. Calculated OPD Wavefront Error RMS for Off-axis Light and the Strehl Ratio**

<table>
<thead>
<tr>
<th>Off-axis Angle</th>
<th>10°</th>
<th>-10°</th>
<th>20°</th>
<th>-20°</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wavefront error RMS ((\lambda))</td>
<td>0.098</td>
<td>0.103</td>
<td>0.2219</td>
<td>0.2385</td>
</tr>
<tr>
<td>Strehl ratio (%)</td>
<td>91.33</td>
<td>83.88</td>
<td>71.32</td>
<td>46.17</td>
</tr>
</tbody>
</table>

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As an example, instead of having a single LC lens with 10 μm thickness and focal length \( f = 400 \text{ mm} \) (10 steps per wave), we use dual LC lenses with an antiparallel rubbing direction (same electrode pattern, but 5 μm thick for each) [Fig. 5(a)], and we calculate the optical performance with 20° off-axis light in the \( x-z \) plane and compare it to the results when two lenses have a parallel rubbing direction in the same off-axis light condition (two lenses with parallel rubbing in stack is optically equal to a single lens with a thickness of 10 μm) [Fig. 5(b)].

The calculation results demonstrate that dual lenses with the antiparallel rubbing arrangement have much better performance than the case of parallel rubbing or a single 10 μm cell at the same oblique incident light. The Strehl ratio is improved to over 80%, normalized to the peak intensity for the ideal lens \( f = 400 \text{ mm} \) in an on-axis light condition. MTF is greatly improved as well. Also, as the phase error is compensated, the results for the antiparallel rubbing arrangement of two lenses have no difference between the positive and negative angle of the incident light with the same magnitude (Fig. 6). Therefore, the off-axis performance of the LC lenses is much improved with this method.

4. Method with Phase Resets in Single Lens

In diffractive optics, phase resets are very useful in increasing the phase ramp while keeping the structure thin. According to Swanson’s work, if the peak-to-peak phase difference between neighboring subzones is multiple times \( 2\pi \) radians, a high efficiency can be achieved [29]. In the case of ideal blazed grating, the phase depth \( \Phi = 2\pi \) radians (one wave); the diffraction efficiency reaches 100% in the first diffraction order, and it is the reason why
diffractive lenses always have one-wave phase depth in each zone. Therefore, if the design wavelength is known, a high efficiency can be obtained with a proper thickness to give a $2\pi$ phase depth. However, with the design with one-wave phase depth for design wavelength, the performance could be significantly different for other wavelengths of light; both the diffraction pattern and the efficiency are affected, lower efficiencies can be expected, and the angle of diffraction is changed as well. Clearly, for nondesign wavelengths of light, the performance deviates from the design wavelength.

Obviously, a large number of resets can greatly shorten the focal length because it decreases the period of the resets, giving a large diffraction angle, and the efficiency for the design wavelength is 100% if peak-to-peak phase difference is $2\pi$. However, for other wavelengths of light, more deviation of the diffraction angle and the efficiency can be expected, and the performance for the visible light spectrum can be greatly degraded for applications such as cameras. Therefore, a higher efficiency can be expected with fewer resets, and adding one or two resets becomes very practical to increase the optical power while keeping the efficiency high, especially for nondesign wavelengths of the light. As a result, it is interesting to investigate the effects of the resets more accurately with the numerical calculation. The variable factors are number of resets, magnitude of phase shift in the reset, etc.

A. Phase Shift Magnitude at the Reset

An example is calculated and shown below: as the OPD for a refractive lens ($d = 2.4$ mm, $f = 400$ mm) from center to edge is about $3.3\lambda$, to double the optical power to $f = 200$ mm with 1 reset, the OPD in each subzone becomes $3.3\lambda$. For the purpose of high efficiency, the phase difference at the reset vicinity between two subzones needs to be integer number of waves in phase; here, one phase profile has $3\lambda$ (design wavelength $\lambda = 543.5$ nm) peak-to-peak phase shift at the reset (Fig. 7, red dotted line); the other one has $1\lambda$ phase shift ($\lambda = 543.5$ nm) (Fig. 7, red solid line). Light distribution at $z = 200$ mm for both cases is calculated as the same as refractive lens $f = 200$ mm.

However, for red and blue light (typically red $\lambda = 610$ nm, blue $\lambda = 470$ nm), the Strehl ratio for the phase profile of the three-wave phase shift drops to about 10% compared to that for the refractive lens with design wavelength of light; the Strehl ratio for the one with the one-wave shift is about 80%. Therefore, in order to have balanced high efficiency for all red, green, and blue light, the phase shift at the reset needs to be $1\lambda$.

B. Effect of Number of Resets

The following calculations are focused on the effect of number of resets more specifically. To design a lens with focal length $f = 440$ mm, the total OPD can be calculated as three waves ($\lambda = 543.5$ nm). If there are two subzones, in each zone there are $1.5\lambda$ and the phase shift at the reset is $1\lambda$ by shifting the center

![Fig. 7. Fresnel lens with $1\lambda$ and $3\lambda$ peak-to-peak phase difference in the reset.](image)

![Fig. 8. Design of Fresnel lens $f = 440$ mm (total OPD 3 waves $\lambda = 543.5$ nm) with two and three subzones.](image)

![Fig. 9. Calculated spot profile at $z = 440$ mm for Fresnel lens with two and three subzones with different wavelengths of light, compared with ideal refractive lens $f = 440$ mm.](image)
zone up by 0.5\%; if there are three subzones, in each zone there is 1\% (Fig. 8). With the phase profile, light distribution at $z = 440$ mm and MTF are calculated for both designs with different wavelengths of light (Figs. 9 and 10).

For green light, the PSF and MTF for both designs are the same as ideal refractive lens, i.e., the efficiency for green is 100\%. However, for red and blue light, the efficiency of the Fresnel design with three subzones drops significantly compared to the design with two subzones. Specifically, for blue, the Strehl ratios of two- and three-subzone Fresnel with respect to ideal refractive lens are 360/475 = 75.8\% and 225/475 = 47.4\%; for red, the Strehl ratios of two- and three-subzone Fresnel with respect to ideal refractive lens are 250/280 = 89.3\% and 200/280 = 71.4\%. Obviously, blue has the most chromatic issues.

These results tend to indicate that the addition of the reset potentially increases the optical power, but at the expense of lower efficiency for blue and red. The regular diffractive lens (1\% at each zone, more than three or four zones) has much more severe chromatic aberrations.

5. Conclusions
The multicell approach and phase reset method are introduced to improve the switching speed, optical power, and the off-axis imaging performance for tunable LC lenses. The problems of LC lenses associated with off-axis light are discussed in detail and a method with a special arrangement of dual lens stacking is proposed. The expected efficiency loss for a stack of lenses due to the decrease of the sampling rate is compensated for with a slightly different design of the electrodes of each cell, which effectively keeps a high sampling rate for the stack. The exact electrode pattern for each component lens is defined clearly with expressions, which can be used straightforwardly in future stack lens designs. Calculations show that the multicell composite lens method provides improvement of the optical power, speed, viewing angle, and optical efficiency. Phase resets can also be used to increase the optical power while keeping fast speed, but at the expense of lower efficiency of non-design wavelengths of light when the number of resets increases.

References